Assignment #7 **Futures, Options and other Derivatives**

10.15. A stock price is currently \$40. Over each of the next two three-month periods it is expected to go up by 10% or down by 10%. The risk-free interest rate is 12% per annum with continuous compounding.

a. What is the value of a six-month European put option with a strike price of \$42?

b. What is the value of a six-month American put option with a strike price of \$42?

10.18. A stock price is currently \$30. Each month for the next two months it is expected to increase by 8% or reduce by 10%. The risk-free interest rate is 5%. Use a two-step tree to calculate the value of a derivative that pays off $\max[(30 - S_T)^2, 0]$, where S_T is the stock price in two months? If the derivative is American-style, should it be exercised early?

10.19. Consider a European call option on a non-dividend-paying stock where the stock price is \$40, the strike price is \$40, the risk-free rate is 4% per annum, the volatility is 30% per annum, and the time to maturity is six months.

a. Calculate u, d, and p for a two-step tree.

b. Value the option using a two-step tree.

c. Verify that DerivaGem gives the same answer.

d. Use DerivaGem to value the option with 5, 50, 100, and 500 time steps.

11.13. A company's cash position (in millions of dollars) follows a generalized Wiener process with a drift rate of 0.1 per month and a variance rate of 0.16 per month. The initial cash position is 2.0.

a. What are the probability distributions of the cash position after one month, six months, and one year?

b. What are the probabilities of a negative cash position at the end of six months and one year?

c. At what time in the future is the probability of a negative cash position greatest?

11.14. Suppose that x is the yield on a perpetual government bond that pays interest at the rate of \$1 per annum. Assume that x is expressed with continuous compounding, that interest is paid continuously on the bond, and that x follows the process

$$dx = a(x_0 - x)\,dt + sx\,dz$$

where a, x_0, and s are positive constants and dz is a Wiener process. What is the process followed by the bond price? What is the expected instantaneous return (including interest and capital gains) to the holder of the bond?