1.28 The correct forward price is $F_0 = S_0\,(1 + r) = 500\,(1 + 0.10) = 550$. As the forward price currently is \$700, the arbitrageur should

1. borrow \$500 at the 10% rate
2. buy one ounce of gold at \$500
3. take a short position on the one-year forward contract with delivery price \$700
4. sell the ounce of gold one year later for \$700
5. pay loan at $\$550 = 500\,(1 + 0.10)$

1.29 The bullish investor has two alternatives: A and B. The option A involves buying 100 shares of the stock at $S_0 = \$94$. The option B consists of buying 2,000 3-month call option with strike price of $X = \$95$ for $C = \$4.7$. Note that both strategies require an initial investment of \$9,400. The profits Π_A and Π_B then are

$$\begin{aligned}
\Pi_A &= \text{number of stock shares} \times (\text{price at maturity} - \text{price today}) \\
&= 100\,(S_3 - 94) \;=\; 100\,S_3 - 9400 \\
\Pi_B &= \text{number of option shares} \times [\max(\text{price at maturity} - \text{strike price}, 0) - \text{option price}] \\
&= 2000\,[\max(S_3 - 95, 0) - 4.7] \;=\; 2000\,\max(S_3 - 95, 0) - 9400
\end{aligned}$$

It rests to find what is the variation Δ in price that implies $\Pi_A = \Pi_B$:

$$\begin{aligned}
\Pi_A = \Pi_B \;\Leftrightarrow\quad & 100\,S_3 = 2000\,\max(S_3 - 95, 0) \\
& 100\,(S_0 + \Delta) = 2000\,\max(S_0 + \Delta - 95, 0) \\
& 9400 + 100\,\Delta = 2000\,\max(94 + \Delta - 95, 0) \\
& 100\,\Delta = 2000\,\max(\Delta - 1, 0) - 9400 \\
& \Delta = 20\,\max(\Delta - 1, 0) - 94 \\
& \Delta = 20\,\max(\Delta, 1) - 114 \\
& \Delta = \max(20\,\Delta, 20) - 114 \\
& \max(20\,\Delta, 20) - \Delta = 114 \\
& \max(19\,\Delta, 20 - \Delta) = 114.
\end{aligned}$$

We must now analyze two cases. In the first case, $\Delta > 1$ and hence

$$\begin{aligned}\Pi_A = \Pi_B \;\Leftrightarrow\quad & \max(19\,\Delta, 20-\Delta) = 114\\ & 19\,\Delta = 114\\ & \Delta = \frac{114}{19} = 6.\end{aligned}$$

In the second case, $\Delta < 1$ and thus

$$\begin{aligned}\Pi_A = \Pi_B \;\Leftrightarrow\quad & \max(19\,\Delta, 20-\Delta) = 114\\ & 20-\Delta = 114\\ & \Delta = 20-114 = -94,\end{aligned}$$

which implies that the stock value is zero three months from now. As for the advice, the choice between the two investments depends on the investor's attitude towards risk. The strategy based on call options offer higher returns than the strategy based on stocks as soon as the price exceeds \$100. However, the same leverage that implies higher returns, also induce higher risk. Indeed, if the price of the stock declines, the stock-based strategy entails a smaller loss than the options-based strategy provided that the stock does not become dust.

1.30 Let Π denote the payoff of the standard oil's bond. The holder receives no interest and, at the maturity, receives \$1,000 plus an additional amount that equals the product of 170 and the excess (if any) of the oil price S_T over \$25 up to a maximum of \$2,550 (corresponds to $S_T = 40$). So,

$$\begin{aligned}\Pi &= 1000 + 170\ \max[\min(S_T, 40) - 25, 0]\\ &= 1000 + 170\ \max[\min(40 - S_T, 0) + S_T - 25, 0]\\ &= 1000 + 170\left\{\max[S_T - 25, -\min(40 - S_T, 0)] + \min(40 - S_T, 0)\right\}\\ &= 1000 + 170\left\{\max[S_T - 25, \max(S_T - 40, 0)] + \min(40 - S_T, 0)\right\}\\ &= \underbrace{1000}_{(B)} + 170\left[\underbrace{\max(S_T - 25, 0)}_{(LC)} + \underbrace{\min(40 - S_T, 0)}_{(SC)}\right].\end{aligned}$$

The decomposition in the terms of (B), (LC) and (SC) clarifies that the standard oil bond's combines the payoffs of a regular bond, a long position on a call option with strike price of \$25, and a short position on a call option with strike price of \$40, respectively.

Remark: One could also start with the more straightforward, though equivalent, characterization of the payoff

$$\Pi = 1000 + \min[2550, 170\ \max(S_T - 25, 0)]$$

and then do the algebra to get the same result. Alternatively, one could do a graphical analysis.

1.32 One must consider the payoffs of the short and long positions on the forward contract. The sequence of actions for the short position reads

1. borrow \$250 at the 6% rate
2. buy one ounce of gold at \$250
3. take a short position on the one-year forward contract with delivery price F_s
4. sell the ounce of gold one year later for $\$F_s$
5. pay loan at $\$265 = 250\,(1 + 0.06)$.

The zero profit condition thus ensures that $F_s = 265$. For the long position, we have that

1. sell one ounce of gold at \$249
2. invest \$249 at the 5.5% rate
3. take a long position on the one-year forward contract with delivery price F_ℓ
4. buy one ounce of gold one year later for $\$F_\ell$

From the zero profit condition, it follows that $F_\ell = 249\,(1 + 0.055) = 262.695$ and hence there is no arbitrage opportunity if the one-year forward price lies in the interval [262.695, 265].

1.34 The payoff of selling an European put option with strike price X is $\min(S_T - X, 0)$. Th payoff of buying an European call option with strike price X is $\max(S_T - X, 0)$. The payoff of th trader that buys a call option and sells a put option with the same underlying asset, strike pric and maturity then is $\Pi = \max(S_T - X, 0) + \min(S_T - X, 0) = S_T - X$. This means that such strategy gives the same payoff from a long position on a forward contract with delivery price X and so it must involve no initial investment as long as the forward price F equals X. This implie that the call and put options will have the same price as long as $X = F$ and the usual pricin conditions for forward contracts hold:

1. there are no transaction costs;
2. the borrowing and lending rates are identical;
3. the tax schemes for the forward and option contracts are identical;
4. there exist arbitrageurs.