2.26 The wheat futures contract has delivery price \$2.50 and size 5,000 bushels, whereas the initial and maintenance margins are \$3000 and \$2000, respectively. This means that the futures price may increase up to

$$\text{current price} + \frac{\text{initial margin} - \text{maintenance margin}}{\text{futures contract size}} = 2.50 + \frac{3000 - 2000}{5000} = 2.70$$

without triggering a margin call. The firm has the right to withdraw any balance in the margin account in excess to the initial margin, hence the futures price must drop by

$$\frac{\text{excess margin}}{\text{futures contract size}} = \frac{1500}{5000} = 0.30$$

so that the excess margin amounts to \$1500.

2.28 As the price of the May 2002 contract exceeds the price of the May 2001 contract, one could engage in the following sequence of actions:

1. March 2001: go long on the May 2001 contract and short on the May 2002 contract;
2. May 2001: buy 5,000 bushels of corn for a price between \$2.1005 and \$2.17075;
3. May 2002: pay the storage costs and sell the corn for a price between \$2.5305 and \$2.59.

To compute the payoff of such a sequence of actions, one must know the exact prices of the May 2001 and May 2002 contracts. For the sake of conservativeness, I will compute the minimum payoff per bushel using the highest price for the May 2001 contract and the lowest price for the May 2002 contract:

$$\Pi_a = 2.5305 - 0.20 - 2.17075\,(1 + 0.05) = 0.0512$$

or

$$\Pi_c = 2.5305 - 0.20 - 2.17075\,e^{0.05} = 0.0485$$

depending on whether the risk-free rate is annually or continuously compounded, respectively. It then suffices to multiply by 5,000 to obtain the total payoff.

2.29 The first step is to compute the series of price changes for both futures contract: $\Delta P_t = P_t - P_{t-1}$. Simple calculations show that the average price changes for the crude oil and gold futures contract are about zero (0.0021 and -0.0625, respectively), supporting the assumption of zero mean, whereas their sample standard deviations are 0.3106 and 2.7715, respectively. To compute the probability p of the event we are interested in, appreciate that, since there is a margin call in the day after, the first price change must be negative and the margin account balance will be back to the initial margin. The second price change must then be such that

$$\text{contract size} \times \text{second price change} < -\text{initial margin}.$$

Now, the maintenance margin is usually about 75% of the initial margin, hence

$$\text{second price change} < -\frac{4}{3}\,\frac{\text{maintenance margin}}{\text{contract size}}.$$

This means that

$$\begin{aligned} p &= \text{Prob}\,(\text{first price change} < 0) \times \text{Prob}\left(\text{second price change} < -\frac{4}{3}\,\frac{\text{maintenance margin}}{\text{contract size}}\right) \\ &= \frac{1}{2}\,\Phi\left(-\frac{4}{3}\,\frac{\text{maintenance margin}}{\text{contract size} \times \text{standard deviation}}\right), \end{aligned}$$

where Φ denotes the cumulative distribution function of the standard normal distribution. It then suffices to obtain the maintenance margin such that

$$\Phi\left(-\frac{4}{3}\,\frac{\text{maintenance margin}}{\text{contract size} \times \text{standard deviation}}\right) = 2\,p$$

or, equivalently, such that

$$\text{maintenance margin} = -\frac{3}{4}\,z_{2p} \times \text{contract size} \times \text{standard deviation}$$

where z_{2p} is the $2p$-quantile of the standard normal distribution. The corresponding quantiles of the standard normal distribution are $z_{0.02} = -2.0537$ and $z_{0.002} = -2.8782$. Given that the contract sizes are 1,000 bbls for the crude oil futures and 100 ounces for the gold futures, the maintenance margins for the crude oil and gold futures must be \$478.42 and \$426.89 per contract at the 1% chance, and \$670.46 and \$598.25 per contract at the 0.1% chance.

3.24 The price of the six-month forward contract is $F_0 = (S_0 - I)\,e^{r/2}$, where $S_0 = \$50$ is the current price of the stock, $I = \$1 \times \exp\left(-\frac{2}{12}\,r\right) + \$1 \times \exp\left(-\frac{5}{12}\,r\right)$ is the present value of the dividends income, and $r = 0.08$ is the risk-free rate. It thus follows that

$$F_0 = \left[50 - \exp\left(-\frac{2}{12}\,0.08\right) - \exp\left(-\frac{5}{12}\,0.08\right)\right]\,e^{\frac{1}{2}\,0.08} = \$50.0068,$$

whereas the initial value of the contract is, by definition, zero. After three months, if the stock price is $S_3 = \$48$, then the price of the forward contract will amount to

$$F_3 = \left[48 - \exp\left(-\frac{2}{12}\,0.08\right)\right]\,e^{\frac{1}{4}\,0.08} = \$47.9630,$$

and the value of the contract is

$$F_0\,e^{-\frac{1}{4}\,r} + I - S_3 = 50.0068\,e^{-\frac{1}{4}\,0.08} + \exp\left(-\frac{2}{12}\,0.08\right) - 48 = \$2.0034$$

given that the forward contract expires in three months and there will be only one dividend payment of \$1 one month before the expiration.

3.25 By borrowing X in cash, the client agrees paying back one year later a cash amount of $X\,(1+r_B)$, where $r_B = 11\%$ is the borrowing rate for cash loans. In contrast, if borrowing the same value X in gold, the client must pay a cash amount of $\frac{X}{S_0}\,(1+r_G)$, where S_0 is the current price of one ounce of gold and $r_G = 2\%$ is the borrowing rate for gold loans. As the risk-free rate is $r = 9.25\%$ and the storage cost is $s = 0.5\%$, the price of the one-year forward contract is $F_0 = S_0\,e^{r+s} = S_0\,e^{0.0975} = 1.1024\,S_0$.

(1) An investor may borrow X in cash today so as to buy X/S_0 ounces of gold and then take a short position on a one-year forward contract with delivery price F_0 and size X/S_0. After one year storing gold, the investor would deliver the gold and pay back the cash loan. The payoff then is

$$\text{(delivery price)} \times \text{(ounces of gold)} - \text{(loan payment)} = F_0\,\frac{X}{S_0} - X(1+r_B) = (1.1024 - 1.11)\,X,$$

resulting in a loss of \$0.0076 X.

(2) Alternatively, the investor may borrow X/S_0 ounces of gold, sell it to obtain X in cash and then invest on the risk-free asset so as to obtain $X\,e^{0.0925} = 1.0969\,X$ in the next year. By taking a long position on the one-year forward contract with delivery price $F_0 = 1.1024\,S_0$ and size $(1+r_G)\,X/S_0$, the investor may buy $(1+r_G)\,X/S_0 = 1.02\,X/S_0$ ounces of gold for F_0 one year later in order to pay the bank loan. The payoff of such a strategy is

$$\text{(principal + interests)} - \text{(due amount of gold)} \times \text{(delivery price)} = 1.0969X - 1.02\,\frac{X}{S_0}\,F_0,$$

which yields a loss of $(1.02 \times 1.1024 - 1.0969)\,X = 0.0276\,X$. We conclude that the borrowing rate for gold is too high relative to the borrowing rate for cash.

(3) Assume now that gold investments yield a $q = 1.5\%$ income per annum. The price of the one-year forward contract then is $F_0 = S_0\,e^{r-q+s} = S_0\,e^{0.0825} = 1.0860\,S_0$. The borrowing rate for cash would then become too high relative to the borrowing rate for gold given that the first and second strategies would yield losses of $0.0240\,X$ and of $(1.02 \times 1.0860 - 1.0969)\,X = 0.0108\,X$, respectively.

3.26 The contract that the firm wishes to agree upon gives it the right to chose the exact delivery date. Suppose that the firm wishes to take a short position. If the domestic risk-free rate is greater than the corresponding foreign risk-free rate, the futures price of the exchange rate increases with the time to maturity. The firm will therefore prefer to deliver as soon as possible and the bank should thus calculate the futures price on the basis that delivery will take place at the beginning of the delivery period. If the domestic risk-free rate is lesser than the corresponding foreign risk-free rate, the futures price of the exchange rate decreases with the time to maturity. The firm will therefore prefer to deliver as late as possible and the bank should thus calculate the futures price on the basis that delivery will take place at the end of the delivery period. The converse is true if the firm wishes to take a long position.

3.27 Trader A enters into a forward contract to buy $1,000,000$ pounds at an exchange rate of 1.6 in three months. Trader B takes a long position on 16 three-month futures contracts with size 62,500 pounds and price 1.6. If the futures price increases to 1.6040, then the profit of trader B is $\Pi_B = 16 \times 62,500 \times (1.6040 - 1.60) = 0.004 \times 10^6 = 4,000$. As for trader A, the new value of the forward contract is $f = 1,000,000 \times (1.6040 - 1.60)\, e^{-\frac{3}{12} r} = 4,000\, e^{-r/4}$. It thus follows that $4,000\, e^{-r/4} = 3,900$ and hence the continuously compounded risk free rate is 10.13% per annum.

3.28 Because the value of the first forward contract is zero, it holds that $K_1 = S_0\, e^{(r-r_f)T_1}$. Settlement of the forward contract at T_1 means that the firm must pay $S_1 - K_1$ to the bank. We therefore must adjust the delivery price K_2 of the second short forward contract, so that it has a value of $K_1 - S_1$ to the firm. By definition, the value of the second short forward contract is $f = (K_2 - F_1)\, e^{-r\,(T_2-T_1)}$, where $F_1 = S_1\, e^{(r-r_f)\,(T_2-T_1)}$ and hence

$$\begin{aligned} K_1 - S_1 &= (K_2 - F_1)\, e^{-r\,(T_2-T_1)} \\ &= K_2\, e^{-r\,(T_2-T_1)} - S_1\, e^{r_f\,(T_2-T_1)}, \end{aligned}$$

which implies that $K_2 = S_1\, e^{(r-r_f)(T_2-T_1)} - (S_1 - K_1)\, e^{r(T_2-T_1)}$.