5.33 The price of the 91-day Eurodollar futures contract that matures in 91 days is \$89.5, which implies that the Eurodollar futures rate is $r_{91}^F = 10.5\%$ per annum with 91-day compounding. To convert the rate into continuous compounding, we must also take the actual/360 day counting convention into account. This yields

$$\begin{aligned} r_0^F &= \frac{365}{91}\log\left(1+\frac{91}{365}\frac{365}{360}r_{91}^F\right) \\ &= \frac{365}{91}\log\left(1+\frac{91}{360}0.105\right) = 0.10507 \end{aligned}$$

with continuous compounding, whereas the 91-day forward rate beginning in 91 days is

$$F_{182|91} = \frac{182\,r_{182} - 91\,r_{91}}{182-91} = 2\,r_{182} - r_{91} = 0.104$$

given that the 91-day and 182-day rates are $r_{91} = 10\%$ and $r_{182} = 10.2\%$, respectively. The arbitrage opportunity is apparent. It suffices to borrow L for 182 days at the rate of 10.2% and then invest L for 91 days at the rate of 10% and take a long position of size $L\,\exp\left(0.10\,\frac{91}{365}\right)$ on the 91-day Eurodollar futures contract that matures in 91 days. As a result, the investor would have a net profit of L times

$$\exp\left(0.10\,\frac{91}{365} + 0.10507\,\frac{91}{365}\right) - \exp\left(0.102\,\frac{182}{365}\right) = 0.000281.$$

To rule out arbitrage opportunities, the futures rate must amount to 10.4% with continuous compounding, and hence the 91-day Eurodollar futures contract maturing in 91 days would have to trade at a higher price, namely, \$89.6083263.

5.35 The value of portfolio A is

$$V_A = 2,000\,\exp(-0.10) + 6,000\,\exp(-0.10\times 10) = 4016.951483,$$

whereas its duration reads

$$D_A = \frac{2,000\,\exp(-0.10) + 10\times 6,000\,\exp(-0.10\times 10)}{2,000\,\exp(-0.10) + 6,000\,\exp(-0.10\times 10)} = 5.945414429.$$

As the portfolio B consists only of a zero-coupon bond, its duration matches the maturity of that bond, i.e., 5.95 years. We therefore conclude that the portfolios have the same duration. This means that small increases in the yield should entail a very similar percentage change in the value of both portfolios. For instance, if the yields increases by 0.1% per annum, then the value of portfolio A becomes

$$V_A' = 2,000\,\exp(-0.101) + 6,000\,\exp(-0.101\times 10) = 3993.179943,$$

which implies a percentage change of -0.591780608%. As for portfolio B, its value changes from

$$V_B = 5,000\ \exp(-0.10 \times 5.95) = 2757.812829,$$

to

$$V'_B = 5,000\ \exp(-0.101 \times 5.95) = 2741.452563,$$

so that the percentage change is -0.593233381%. However, this result does not hold for large variations in the yields. For instance, if the yields increase by 5%, the values of portfolios A and B respectively become

$$\begin{aligned} V''_A &= 2,000\ \exp(-0.15) + 6,000\ \exp(-0.15 \times 10) = 3060.196914 \\ V''_B &= 5,000\ \exp(-0.15 \times 5.95) = 2048.151978 \end{aligned}$$

so that the percentage changes are not so similar, namely, -23.81792694% and -25.73274168%, respectively. The fact that the value of portfolio A decreases by a lesser amount than the value of portfolio B reflects the greater convexity of portfolio A.

5.36 As the price of the 6-month zero-coupon bond is \$98, the 6-month zero rate with continuous compounding is $2\ \log\left(1 + \frac{100-98}{98}\right) = 4.0405415\%$. Similarly, the 12-month zero rate with continuous compounding is $\log\left(1 + \frac{100-95}{95}\right) = 5.1293294\%$. As for the 18-month zero rate, it holds that

$$\frac{6.2}{2}\, e^{-\frac{1}{2}0.040405415} + \frac{6.2}{2}\, e^{-0.051293294} + \left(100 + \frac{6.2}{2}\right)\, e^{-\frac{3}{2}r_{18}} = 101,$$

which implies that $r_{18} = 5.4429045\%$ per annum with continuous compounding. As for the 24-month zero rate, it ensues from

$$\frac{8}{2}e^{-\frac{1}{2}0.040405415} + \frac{8}{2}e^{-0.051293294} + \frac{8}{2}e^{-\frac{3}{2}0.054429045} + \left(100 + \frac{8}{2}\right)\, e^{-2r_{24}} = 104,$$

that $r_{24} = 5.8085446\%$ per annum with continuous compounding. We now compute the 6-month forward rates using

$$r_i^F = \frac{(i+6)\, r_{i+1} - i\, r_i}{6}, \qquad i = 12, 18, 24.$$

This results in $r_{12}^F = 6.2181174\%$, $r_{18}^F = 6.0700547\%$, and $r_{24}^F = 6.9054646\%$. As for the par yield c, we know that, for semiannual coupon payments,

$$c = 2\,\frac{1-d}{A},$$

where d is the present value of \$1 at the maturity of the bond and A is the value of the annuity that pays \$1 at each coupon payment date. It thus follows that

$$c_6 = 2\,\frac{1 - e^{-\frac{1}{2}0.040405415}}{e^{-\frac{1}{2}0.040405415}} = 4.081632653\%$$

$$\begin{aligned}
c_{12} &= 2\,\frac{1-e^{-0.051293294}}{e^{-\frac{1}{2}0.040405415}+e^{-0.051293294}} = 5.18134715\% \\
c_{18} &= 2\,\frac{1-e^{-\frac{3}{2}0.054429045}}{e^{-\frac{1}{2}0.040405415}+e^{-0.051293294}+e^{-\frac{3}{2}0.054429045}} = 5.498639456\% \\
c_{24} &= 2\,\frac{1-e^{-2\times 0.058085446}}{e^{-\frac{1}{2}0.040405415}+e^{-0.051293294}+e^{-\frac{3}{2}0.054429045}+e^{-2\times 0.058085446}} = 5.86206209\%.
\end{aligned}$$

Finally, the price of a 24-month bond providing a semiannual coupon of 7$ per annual is

$$\frac{7}{2}e^{-\frac{1}{2}0.040405415}+\frac{7}{2}e^{-0.051293294}+\frac{7}{2}e^{-\frac{3}{2}0.054429045}+\left(100+\frac{7}{2}\right)e^{-2\times 0.058085446} = 102.1290383,$$

with yield $y = 0.057723217$ so as to satisfy

$$\frac{7}{2}e^{-\frac{1}{2}y}+\frac{7}{2}e^{-y}+\frac{7}{2}e^{-\frac{3}{2}y}+\left(100+\frac{7}{2}\right)e^{-2y} = 102.1290383.$$

5.38 The number of futures contracts that one must short is the nearest integer to

$$\frac{\text{portfolio duration}\times\text{portfolio value}}{\text{bond duration}\times\text{futures price}\times\text{contract size}} = \frac{4\times 100,000,000}{9\times 122\times 100,000} = 3.64298725,$$

hence 4 contracts. If the duration of the cheapest-to-deliver bond were 7 years, then the investo would have to short 5 contracts given that $\frac{4\times 100,000,000}{7\times 122\times 100,000} = 4.683840749$. It thus follows tha the investor must short one additional futures contract. If the interest rates increase over th three months in such manner that the term structure becomes less steep, then the hedge woul underperform because the increase in the value of the short futures contract would be lower tha the increase in the value of the bond portfolio due to the former's longer duration.