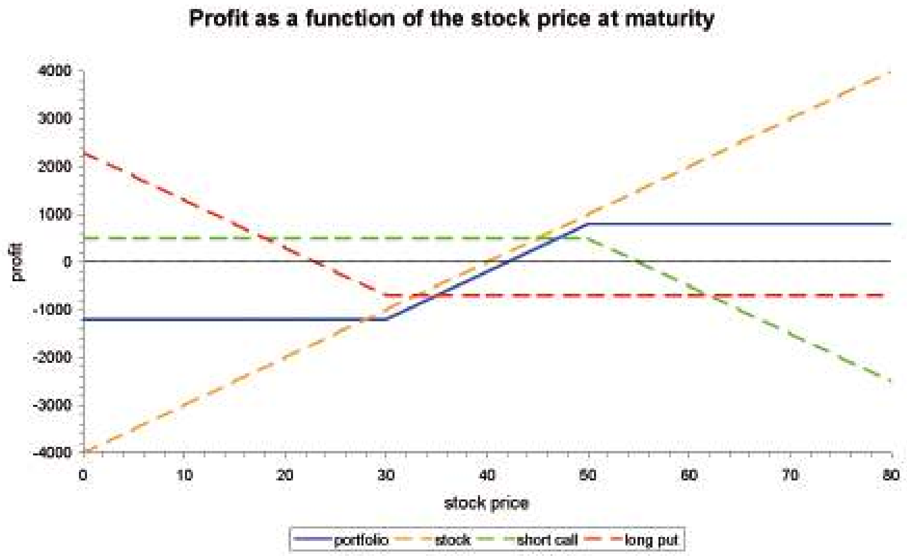
**7.16** The payoff of an investor who detains 100 shares, a short position on 100 call options with strike price of \$50, and a long position on 100 put options with strike price of \$30 is

$$\begin{aligned} 100\left[S_T - \max(S_T - 50, 0) + \max(30 - S_T, 0)\right] &= 100\left[S_T + \min(50 - S_T, 0) + \max(30 - S_T, 0)\right] \\ &= 100\left[\min(50, S_T) + \max(30, S_T) - S_T\right]. \end{aligned}$$

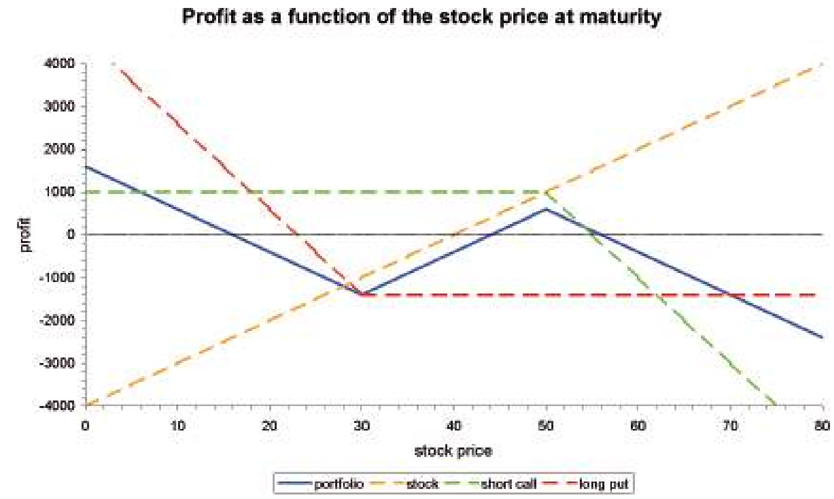
In view that such a portfolio costs $100\,(S_0 - c + p) = 100\,(40 - 5 + 7) = 4,200$. The investor's profit then is

$$100\left[\min(50, S_T) + \max(30, S_T) - S_T\right] - 4,200 = 100\left[\min(50, S_T) + \max(S_T, 30) - S_T - 42\right].$$

It therefore follows that, if $S_T \leq 30$, the investor loses \$1,200 $= 100\,(30 - 42)$. If $S_T \geq 50$, then the investor gains \$800 $= 100\,(50 - 42)$. Finally, if $30 < S_T < 50$, the investor's profit is $100\,(S_T - 42)$. The first diagram plots the profit as a function of the stock price at maturity.




We now consider the case in which the investor holds 100 shares, a short position on 200 call options with strike price of \$50, and a long position on 200 put options with strike price of \$30. The portfolio entails a payoff of $100\left[2\,\min(50, S_T) + 2\,\max(30, S_T) - 3\,S_T\right]$ with a cost of $100\,(S_0 - 2\,c + 2\,p) = 4,400$. The resulting payoff is $100\left[2\,\min(50, S_T) + 2\,\max(30, S_T) - 3\,S_T\right]$ as illustrated in the second diagram.



**8.21** It suffices to apply the put-call parity to see that there is a arbitrage opportunity. More precisely, the put-call parity dictates that the value of the put option must be

$$3 + 20\,\exp\left(-0.10\,\frac{3}{12}\right) + \exp\left(-0.10\,\frac{1}{12}\right) - 19 = \$4.50,$$

which is greater than the current price of \$3. As the put is undervalued relative to the call option, the arbitrageur would buy the stock and the put, and short the call.

**8.22** Consider a portfolio that consists of a long call with strike price $K_1$, a long call with strike price $K_3$ and a short call with strike price $K_2$, where $K_2 - K_1 = K_3 - K_2 > 0$. The payoff $\Pi$ of such a portfolio then is $\max(S_T - K_1, 0) + \max(S_T - K_3, 0) - \max(S_T - K_2, 0)$ or, equivalently,

$$\Pi = \begin{cases} 0 & \text{if} \quad S_T \leq K_1 \\ S_T - K_1 & \text{if} \quad K_1 < S_T \leq K_2 \\ (K_2 - K_1) - (S_T - K_2) & \text{if} \quad K_2 < S_T \leq K_3 \\ (K_2 - K_1) - (K_3 - K_2) & \text{if} \quad S_T > K_3. \end{cases}$$

It turns out, however, that $(K_2 - K_1) - (K_3 - K_2) = 0$ and $(K_2 - K_1) - (S_T - K_2)$ is nonnegative given that $S_T - K_2 \leq K_3 - K_2 = K_2 - K_1 > 0$. As the portfolio always provides a nonnegative payoff at the expiration of the option, it must have a nonnegative value today and hence $c_1 + c_3 - 2\,c_2 \geq 0$.

**8.23** Consider a portfolio consisting of a long put with strike price $K_1$, a long put with strike price $K_3$ and a short put with strike price $K_2$, where $K_2 - K_1 = K_3 - K_2 > 0$. The payoff $\Pi$ of such a portfolio then is $\max(K_1 - S_T, 0) + \max(K_3 - S_T, 0) - \max(K_2 - S_T, 0)$, i.e.,

$$\Pi = \begin{cases} (K_3 - K_2) - (K_2 - K_1) & \text{if} \quad S_T \leq K_1 \\ (K_3 - K_2) - (K_2 - S_T) & \text{if} \quad K_1 < S_T \leq K_2 \\ K_3 - S_T & \text{if} \quad K_2 < S_T \leq K_3 \\ 0 & \text{if} \quad S_T > K_3. \end{cases}$$

As the portfolio always provides a nonnegative payoff, it must have a nonnegative value today, which implies that $p_1 + p_3 - 2\,p_2 \geq 0$.

**8.24** Denote by $V$ and $D$ the value of the firm and the face value of the debt, respectively. The manager position then corresponds to a call option on $V$ with strike price $D$ given that its payoff is $\max(V - D, 0)$. In contrast, the debtholders get

$$\min(V, D) = D + \min(V - D, 0) = D - \max(D - V, 0),$$

and hence it is as if they were holding a risk-free holding that worths $D$ at maturity and a short put on $V$ with strike price $D$. To increase the value of the manager position, (s)he must increase either the value of firm or its volatility so as to positively affect the value of the call option (s)he is holding.