Should Educational Policies be Regressive?

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Abstract

We show that when the government is able to transfer wealth between generations, regressive policies - as proposed by De Fraja (2002) - are no longer optimal. The optimal educational policy can be decentralized through appropriate Pigouvian taxes and credit provision, is not regressive, and provides equality of opportunities in education (in the sense of irrelevance of parental income for the amount of education).

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1 Introduction

The role of educational policies in equalizing opportunities is a widely accepted issue in political debates. However, a remarkable feature of most educational systems in the world is the huge regressivity of spending per students (i.e., children from wealthier families receive more education than those from poorer families).[1] This regressivity of educational systems may indicate either the presence of some trade-off between equity and efficiency or the inefficiency of observed policies.

The existence of a trade-off between redistribution and efficiency in taxation is known at least since the work of Mirrlees (1971). In the specific case of education, this issue has been previously discussed by Becker (1991) in the context of the parent's decision on the education provided to children with

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[1] See, for example, Bishop (1977), Campbell and Siegel (1967), Fernandez and Rogerson (1996), Kozol (1991), Psacharopoulus (1986), and Radner and Miller (1970).

In the United States, this regressivity is reflected in the large disparity of spending per students across districts. Since 43 percent of elementary and secondary education is financed at the local level, 49.9 percent are financed at the state level, and only 7.1 percent are financed at the federal level (2001 Census of Governments), these differences reproduce the inequality of income distribution. Fernandez and Rogerson (1998) and Inman (1978) provided general equilibrium computations of the welfare gains associated with the centralization of educational expenses.

different abilities. Hare and Ulph (1979) find that the optimal educational policies will be egalitarian (in the sense of constant consumption and utility) only for intermediate abilities.

The theoretical literature on optimal educational policies in an asymmetric information context was pioneered by Ulph (1977) and Hare and Ulph (1979) who extended the optimal taxation approach of Mirrlees (1971) to address the problem of determining the optimal educational and taxation policies jointly when the ability to benefit from education is unobservable. More recently, De Fraja (2002) studied the optimal educational provision in an overlapping-generations model in the presence of externalities and imperfect capital markets. His results suggest that educational policies should be regressive (in the sense that households with brighter children and higher income contribute less than those with less bright children and lower income) and do not provide equality of opportunities in education (in the sense of irrelevance of the education received by a child on household's income). Therefore, the regressivity of educational systems in most countries may actually reflect the optimal educational policies and the provision of equality of opportunities in education may imply a great efficiency loss.

We shall argue that the results obtained at De Fraja (2002) follow from a particular restriction on the government's budget constraint: budget is imposed to be balanced with each generation at any time. Since we are considering an overlapping-generations model, the government would usually be able to transfer between generations. Indeed, this is exactly what pay-as-you-go social-security systems are: young generations contribute for the benefits of the older generations. As most social-security systems in the world are (at least partially) pay-as-you-go systems, it seems reasonable to assume that governments are able to transfer between generations.

We show that if we allow for transfers between generations, then the optimal educational policy takes a very different form: it achieves first-best welfare and provides equality of opportunities at education. Moreover, it can be decentralized through appropriate Pigouvian taxes and the provision of credit.

In the decentralized mechanism, first-best welfare is reached through a subsidy on education to correct for the externalities, a lump sum taxation proportional to the average education and the provision of credit (at the market interest rate). Such a mechanism is not regressive (i.e., wealthier households do not contribute less than poorer households and households with brighter children contribute more than those with less bright children) and does not require knowledge of each household's wealth.

Hence, our results suggest that the observed inequalities must reflect an inefficiency in educational systems. We also propose an intuitive and informationally less demanding educational policy.

There is also a well-established theoretical literature that emphasizes the public choice perspective of public education financing rather than focusing on efficiency arguments. In this literature, it is usually assumed that educational provision must be uniform for each neighborhood and the amount of education is decided through majority voting.[2]

In section 2, we present the basic framework of the model. The structure is the same as De Fraja (2002) except for the government budget constraint. In section 3 we present the laissez-faire equilibrium. In section 4, we present the government intervention solution. In subsection 4.1, we characterize the first-best equilibrium; then we present the second-best equilibrium (4.2) and the decentralized equilibrium (4.3). Section 5 summarizes the main results of the paper.

2 The Basic Framework

As in De Fraja (2002), consider an economy with a continuum of households with measure normalized to 1. Each household consists of a parent and a child. An individual lives for 2 periods. In the first one, she receives an education and a bequest. In the second period, she works, has a child, consumes, and provides an education and a bequest for her daughter.

Each individual's utility function is:

$$U = u(c) + x$$

where c is her consumption and x is the amount of monetary resources available to the child. We assume that $u \in C^2$ satisfies: [3]

$$u'(c) > 0,\ u''(c) < 0,\ \lim_{c \to 0} u'(c) = +\infty \text{ and } u'(c^*) = 1 \text{ for some } c^* \in \Re_{++}$$

There are two ways of transferring wealth to the child: bequest t and higher future wages (through education e). We normalize the interest rate paid on bequests to 1. The technology that transforms education in future wages is $y(\theta, e; E)$, where $\theta \in [\theta_0, \theta_1]$ is each child's productivity parameter, e is the amount of education and E is the general level of education. We assume that $y \in C^2$ satisfies:

$$\begin{aligned} y_e(\theta, e; E) &> 0,\ y_\theta(\theta, e; E) > 0,\ y_{e\theta}(\theta, e; E) > 0 \\ y_E(\theta, e; E) &> 0,\ y_{ee}(\theta, e; E) < 0,\ \lim_{e \to 0} y_e(\theta, e; E) = +\infty \end{aligned}$$

The assumption $y_E(\theta, e; E) > 0$ means that education is a source of a positive externalities.[4] This is the most interesting case in the model presented although the results trivially hold when there is no externality in education

[2] This strand of the literature includes De Bartolome (1990), Epple and Romano (1996), Fernandez and Rogerson (1995, 1996, 1998), Glomm and Ravikumar (1992), Johnson (1984), Peltzman (1973), and Stiglitz (1974).

[3] As De Fraja (2002) noted, the dependence of the parent´s utility function on the child´s wealth rather than on her utility is a usual assumption and greatly simplifies the analysis. Because of its linearity, it implies that the mother is risk neutral in the wealth left to her daughter.

[4] See Acemoglu (1996), Lucas (1988), and Moretti (2003a, 2003b) for discussions on the presence of human capital externalities.

(i.e. $y_E(\theta, e; E) = 0$). The hypothesis $y_{e\theta}(\theta, e; E) > 0$ means that education increases earnings more for abler individuals.

Substituting the two possible ways of transferring wealth to the child, we get:

$$x = y(\theta, e; E) + t \tag{1}$$

Let Y_i be the parent's wealth ($i \in \{1, 2, ..., n\}$) and k be the monetary cost of a unit of education.[5] With no loss of generality, we may assume that $Y_1 \leq Y_2 \leq ... \leq Y_n$. Then, the household's budget constraint is:

$$Y_i = c + ke + t \tag{2}$$

Let $\phi(\theta)$ and h_i be the probability functions of θ and Y_i, respectively. We assume that $\phi(\theta) > 0$ for all $\theta \in [\theta_0, \theta_1]$.

Substituting (1) and (2) in the utility function, it can be written as:

$$U = u(Y_i - t - ke) + y(\theta, e; E) + t$$

3 The Laissez-Faire Equilibrium

The imperfection of educational credit markets was studied, among others, by Becker (1964) and Schultz (1963). It is usually argued that investment in human capital is risky, nondisversifiable, and hard to collateralize implying that private credit markets may fail to finance education. In this economy, credit markets are imperfect in the sense that individuals cannot leave negative bequests.[6] The household's problem is:

$$\max_{\{e,t\}} \; u(Y_i - t - ke) + y(\theta, e; E) + t \qquad s.t. \; t \geq 0$$

Define $e^u(\theta, Y_i, k, E)$ and $e^c(\theta, Y_i, k, E)$ implicitly by the relations:[7]

$$k = y_e(\theta, e^u; E), \quad ku'(Y_i - ke^c) = y_e(\theta, e^c; E)$$

Solving the household's problem we obtain the following proposition:

Proposition 1 *The laissez-faire competitive equilibrium allocation is*

[5] The mother's wealth Y_i is a function of her education which is predetermined in the period we study.

In this model, public and private schools provide education at the same cost implying that the actual provider of education is immaterial. Hence, we abstract from the discussion on education should be privately or publicly provided (see Lott, 1987).

[6] This is a usual assumption in education and child labour models. See, for example, Baland and Robinson (2000) or Ranjan (2001). Benabou (2003) and Glomm and Ravikumar (1992) assume that there is no credit market and education is the only mean of transfering wealth (i.e., $t \equiv 0$).

[7] The letters u and c stand for unconstrained and constrained, respectivelly. From the assumptions on $u(\cdot)$ and $y(\cdot, \cdot; \cdot)$, it´s easy to show the existence of e^u and e^c.

$\{c_i(\theta), e_i(\theta), t_i(\theta);\ \theta \in [\theta_0, \theta_1],\ i = 1, ..., n\}$, *such that:*

$$\begin{aligned} e_i(\theta) &= \max\{e^u(\theta, Y_i, k, E); e^c(\theta, Y_i, k, E)\} \\ t_i(\theta) &= \max\{Y_i - c^* - ke(\theta, Y_i, k, E); 0\} \\ c_i(\theta) &= \min\{c^*; Y_i - ke(\theta, Y_i, k, E)\} \end{aligned}$$

Proof. The first order conditions to the household's problem (necessary and sufficient) are:

$$\begin{aligned} ku'(Y_i - t - ke) &= y_e(\theta, e; E) \\ u'(Y_i - t - ke) &= 1 + \mu \\ \min\{t, \mu\} &= 0 \end{aligned}$$

where μ is the Kuhn-Tucker multiplier. If $\mu = 0$, we say that the solution to the problem is unconstrained and it follows that:

$$\begin{aligned} y_e(\theta, e^u; E) &= k \\ Y_i - c^* - ke^u &= t^u \end{aligned}$$

If $\mu > 0$, we say that the solution to the problem is constrained and it follows that:

$$\begin{aligned} y_e(\theta, e^c; E) &= ku'(Y_i - ke^c) < k \therefore e^c < e^u \\ u'(Y_i - ke^u) &> 1 \therefore c^u < c^* \end{aligned}$$

■

Hence, if $Y_i < c^* + ke^u$, the household's decisions are constrained since the parent would prefer to leave negative bequests but she is not allowed to. Then, she partially reduces her consumption and partially reduces her daughter's education. Since education is increasing in θ, households with sufficiently bright children (high θ) or low wealth Y_i are constrained.[8]

4 The Government Intervention Solution

4.1 The First-Best Solution

As in most public finance literature, we take a government that maximizes the unweighted sum of every individual's utilities. The total level of education is

[8] Applying the implicit function theorem, we get:

$$\begin{aligned} \frac{\partial e^u}{\partial \theta} &= -\frac{y_{e\theta}(\theta, e^u; E)}{y_{ee}(\theta, e^u; E)} > 0 \\ \frac{\partial e^c}{\partial \theta} &= -\frac{y_{e\theta}(\theta, e^c; E)}{y_{ee}(\theta, e^u; E) + k^2 u''(Y_i - ke^c)} > 0 \end{aligned}$$

defined as the sum of each individual's education:[9]

$$E = \int_{\theta_0}^{\theta_1} \sum_{i=1}^{n} h_i e_i(\theta)\,\phi(\theta)\,d\theta \tag{3}$$

Then, the first-best solution (or, equivalently, the symmetric Pareto optimal allocation) is the solution to the following problem:

$$\max_{\{e_i(\theta),t_i(\theta),E\}} \int_{\theta_0}^{\theta_1} \sum_{i=1}^{n} h_i \left[u\left(Y_i - t_i(\theta) - k e_i(\theta)\right) + y\left(\theta, e_i(\theta); E\right) + t_i(\theta)\right]\phi(\theta)\,d\theta$$
$$\text{s.t.} \quad E = \int_{\theta_0}^{\theta_1} \sum_{i=1}^{n} h_i e_i(\theta)\,\phi(\theta)\,d\theta$$

For notational convenience we shall define the expectations operator $E[\cdot]$ as:

$$E[e_i(\theta)] \equiv \int_{\theta_0}^{\theta_1} \sum_{i=1}^{n} h_i e_i(\theta)\,\phi(\theta)\,d\theta$$

Define e_i^* implicitly by the relation:[10]

$$k = y_e\left(\theta, e_i^*(\theta); E[e_i^*(\theta)]\right) + E\left[y_E\left(\theta, e_i^*(\theta), E\left(e_i^*(\theta)\right)\right)\right]$$

Let $t_i^*(\theta)$ be defined as $t_i^*(\theta) = Y_i - k e_i^*(\theta) - c^*$. Assume that $E[t_i^*(\theta)] \geq 0$.[11]

Proposition 2 *The first-best allocations are* $\{c^*, e_i^*(\theta), t_i^*(\theta);\ \theta \in [\theta_0, \theta_1],\ i = 1, ..., n\}$.

Proof. The first order conditions (necessary and sufficient) for the problem above are:

$$\begin{aligned}
\left[-u'\left(Y_i - t_i(\theta) - k e_i(\theta)\right) + 1\right] h_i \phi(\theta) &= 0 \\
\left[-k\ u'\left(Y_i - t_i(\theta) - k e_i(\theta)\right) + y_e\left(\theta, e_i(\theta), E\right) + \lambda\right] h_i \phi(\theta) &= 0 \\
E\left[y_E\left(\theta, e_i(\theta); E\right)\right] - \lambda &= 0
\end{aligned}$$

where λ is the Lagrange multiplier associated to (3). Solving these equations, we get the result above. ■

Notice that the optimal education $e_i^*(\theta)$ is independent of the household's wealth Y_i. Thus, the first-best amount of education is characterized by equality of opportunities in the sense that individuals with the same ability receive the same education (see De Fraja, 2002b).[12] As efficiency requires that marginal

[9] This specification implies in the same amount of externality being produced by any unit of education (i.e., the amount of externality caused by a year in high school is the same as in the PhD). However, as would be clear when we present the decentralized scheme, the main results do not depend on such assumption.

[10] From the concavity of y, it follows that $e_i^*(\theta)$ exists for all θ.

[11] This assumption garantees that there are enough resources so that e_i^* and c^* are feasible in a symmetric information economy.

[12] We also get that the optimal consumption level c^* is independent of Y_i.

productivity of education must be equalized for all individuals, it follows that the amount of education received by an individual should depend only on her ability.

Moreover, positive externalities imply that an inefficiently low amount of education is provided in the laissez-faire competitive equilibrium even for unconstrained households (since y is strictly concave in e).

Because marginal productivity of education is increasing in ability, it follows that education provided in the first-best solution is also increasing in ability (i.e., the first-best equilibrium is input-regressive).[13]

4.2 The Second-Best Equilibrium

Consider a government that can offer a tax schedule and an education schedule. A tax schedule consists of an income tax τ_i. An education schedule consists of an offer of education e, an up-front fee $f_i(\theta)$ and a deferred payment $m_i(\theta)$. Since $f_i(\theta)$ and $m_i(\theta)$ may be positive or negative, the government is able to offer loans to students.[14]

With no loss of generality, we can normalize each household's bequests to zero. In that case, all bequests are left through up-front fees and defered payments. The household's budget constraint is:

$$Y_i \geq c_i(\theta) + \tau_i + f_i(\theta)$$

Substituting in the utility function, it can be written as:

$$U_i(\theta) = u(Y_i - \tau_i - f_i(\theta)) + y(\theta, e_i(\theta); E) - m_i(\theta) \tag{4}$$

From the revelation principle, the search for an optimal educational policy can be restricted to the class of incentive-compatible mechanisms with no loss of generality. As usual, the incentive-compatibility constraint can be rewritten as a local condition using the first-order and second-order conditions and the envelope theorem:

Lemma 3 *A policy* $\{\tau_i, f_i(\theta), m_i(\theta), e_i(\theta); 1 \leq i \leq n, \theta \in [\theta_0, \theta_1]\}$ *is incentive-compatible if, and only if, it satisfies:*

$$\dot{U}_i(\theta) = y_\theta(\theta, e_i(\theta); E) \tag{5}$$

$$\dot{e}_i(\theta) \geq 0 \tag{6}$$

for all $i \in \{1, ..., n\}$, $\theta \in [\theta_0, \theta_1]$

[13] Applying the implicit function theorem, $\frac{\partial e_i^i(\theta)}{\partial \theta} = -\frac{y_{e\theta}}{y_{ee}} > 0$.

[14] By allowing the government to charge deferred payments, we focus on children above some minimum age. As Becker and Murphy (1988) argue, young children usually cannot be a party to these type of contracts.

Proof. See De Fraja (2002) p.446. ■

We also assume that individuals are not forbidden to purchase education in the private sector. Hence, they will only join the educational program when their utility exceeds the utility obtained if they purchase education privately. Define $P(\theta, Y, E)$ as the utility obtained in the laissez-faire equilibrium. Then, the constraint on a household's utility is:

$$U_i(\theta) \geq P(\theta, Y_i - \tau_i, E), \ \ \forall i, \forall \theta$$

Up to this point, our model is exactly like De Fraja (2002). The distinct feature is that we will enable the government to transfer resources between generations. In each period there is a generation of young (paying $\tau_i + f_i(\theta)$ as taxes and receiving $ke_i(\theta)$ in education) and old individuals (paying the deferred payments $m_i(\theta)$) and we allow the government to transfer resources between them. Hence, the government budget constraint is:

$$\int_{\theta_0}^{\theta_1} \sum_{i=1}^{n} h_i \left[\tau_i + f_i(\theta) + m_i(\theta)\right] \phi(\theta)\, d\theta \geq \int_{\theta_0}^{\theta_1} \sum_{i=1}^{n} h_i \left[ke_i(\theta)\right] \phi(\theta)\, d\theta \tag{7}$$

Equation (7) states that the net tax revenue (L.H.S.) is enough to finance the educational expenses (R.H.S.).

The government problem is:

$$\max_{\{e_i(\theta), \tau_i, f_i(\theta), m_i(\theta)\}} \int_{\theta_0}^{\theta_1} \sum_{i=1}^{n} h_i \left[u(Y_i - \tau_i - f_i(\theta)) + y(\theta, e_i(\theta); E) - m_i(\theta)\right] \phi(\theta)\, d\theta$$
$$s.t. \ (3), (4), (5), (6), (7)$$

Solving the government problem, we get the following proposition which proof is presented in the Appendix:

Proposition 4 *The optimal educational policy implements* $\{e_i^*(\theta), c^*;\ \theta \in [\theta_0, \theta_1],\ i = 1, ..., n\}$ *and achieves first-best welfare.*

Thus, when transfers between generations are allowed, the optimal educational policy provides equality of opportunities in education (since $e_i^*(\theta)$ does not depend on Y_i as we have shown in the preceding section). As we have shown, the efficient amount of education is higher than in education provided in the laissez-faire equilibrium. Moreover, contrary to the results of De Fraja (2002), *the amount of education and consumption does not depend on each parent's wealth.*

In the next section, we show that the optimal educational policy can be decentralized through Pigouvian taxes. This decentralization is desirable due to its simplicity and informational advantage.

4.3 The Equilibrium with Pigouvian taxes

Since Pigou (1938), economists know that efficiency in an externality generating activity can be reached through the imposition of Pigouvian taxes.[15] As Carlton and Loury (1980) show, efficiency may require an additional lump sum tax-subsidy scheme. In this section, we show that the optimal mechanism can be decentralized through appropriate Pigouvian taxes and the provision of credit at the market interest rate. Moreover, the decentralized scheme does not require knowledge of household's wealth and seems more consistent with actual educational policies.

Let τ_i, $f_i(\theta)$ and $m_i(\theta)$ be the taxes defined before. Define $t_i(\theta)$, f and $\hat{k}$ as:

$$\begin{aligned} t_i(\theta) &= -m_i(\theta) \\ f_i(\theta) &= t_i(\theta) + \hat{k}e_i(\theta) \\ \tau_i &= f \end{aligned}$$

Hence, we are restricting the mechanism offered to a lump sum tax f, a linear in education up-front fee $t_i(\theta) + \hat{k}e_i(\theta)$ and a deferred payment $-m_i(\theta)$ (which is a subset of the class of contracts considered previously). This mechanism can be alternatively interpreted as a loan $-t_i(\theta)$ and an up-front fee $\hat{k}e_i(\theta)$. Clearly, allowing for loans makes it possible to relax the non-negative bequests constraint.

In the each period, the government pays $\left(k - \hat{k}\right)$ as a subsidy on each unit of education and receives f as a lump sum tax. The government also loans $E[-t_i(\theta)]$ in the first period and receives it in the next period. Since the market interest rate is normalized to 1, $E[-t_i(\theta)]$ may take any value because it's always repaid in the following period. Thus, the government's budget constraint is:

$$\int_{\theta_0}^{\theta_1} \sum_{i=1}^{n} h_i \left[\left(k - \hat{k}\right) e_i(\theta) - f\right] \phi(\theta)\, d\theta \leq 0 \tag{8}$$

The household's budget constraint for this mechanism is:

$$Y_i \geq c_i(\theta) + t_i(\theta) + f + \hat{k}e_i(\theta) \tag{9}$$

Hence, we can write the household's problem as:

$$\max_{\{e_i(\theta), t_i(\theta)\}} u\left(Y_i - t_i(\theta) - f - \hat{k}e_i(\theta)\right) + y(\theta, e_i(\theta); E) + t_i(\theta)$$

Lemma 5 *The solution to the household's problem is* $\{c_i^P(\theta), e_i^P(\theta), t_i^P(\theta);$ $\theta \in [\theta_0, \theta_1],\ i = 1, ..., n\}$, *such that:*[16]

$$c_i^P(\theta) = c^*$$

[15] See Baumol (1972) and Kopczuk (2003).

[16] The concavity of y implies in the existence of $e_i^P(\theta)$.

$$\hat{k} = y_e\left(\theta, e_i^P(\theta); E\right) \tag{11}$$
$$t_i^P(\theta) = Y_i - c^* - f - \hat{k}e_i^P(\theta)$$

Proof. The result follows from the first order conditions of the household's problem. ■

Now, we are ready to show that the first-best solution can be reached through a suitable choice of f and $\hat{k}$. As in Carlton and Loury (1980), the efficient allocation can be reached in this context through Pigouvian subsidies and a lump sum tax. This result can be seen as an application of the so-called 'Principle of Targeting' according to which externalities should be corrected by targeting its source directly.

Proposition 6 *The first-best welfare is reached in the asymmetric information context through an appropriate choice of an up-front fee. Moreover, this equilibrium satisfies the government's budget constraint (8).*

Proof. Set $\hat{k}$ as:

$$\hat{k} = k - E\left[y_E(\theta, e_i^*(\theta), E(e_i^*(\theta)))\right] \tag{12}$$

where $E[x_i(\theta)] \equiv \int_{\theta_0}^{\theta_1} \sum_{i=1}^{n} h_i x_i(\theta)\phi(\theta)\,d\theta$. Substituting in the first order conditions of the household's problem (11), we get $e_i(\theta) = e_i^*(\theta)$.

Set f as:

$$f = E\left[e_i^*(\theta)\right]\ E\left[y_E(\theta, e_i^*(\theta), E\left[e_i^*(\theta)\right])\right] \tag{13}$$

Then, we get:

$$E[t_i(\theta)] = E\left[Y_i - \hat{k}e_i^*(\theta) - c^* - f\right] = E\left[Y_i - ke_i^*(\theta) - c^*\right] = E\left[t_i^*(\theta)\right]$$

Hence, as c^*, $e_i^*(\theta)$, and $E[t_i^*(\theta)]$ are the same as in the first-best solution (and utility is linear in t), first-best welfare is achieved.

From (12), $\left(k - \hat{k}\right)e_i^*(\theta) = E\left[y_E(\theta, e_i^*(\theta), E(e_i^*(\theta)))\right]\ e_i^*(\theta)$. Then, we get:

$$E\left[\left(k - \hat{k}\right)e_i^*(\theta)\right] = E\left[y_E(\theta, e_i^*(\theta), E(e_i^*(\theta)))\right]\ E\left[e_i^*(\theta)\right] \tag{14}$$

Hence, (13) and (14) implies that $E\left[\left(k - \hat{k}\right)e_i^*(\theta) - f\right] = 0$. It follows that the government's budget constraint (8) is satisfied. ■

As $E[t_i(\theta)] = E[t_i^*(\theta)] > 0$, the government transfers resources from older individuals to younger individual (who repay when older). Define the household's financial contribution as:

$$z_i(\theta) \equiv f + \hat{k}e_i(\theta)$$

Since education is independent of wealth, it's clear that an individual's financial contribution is independent of her income. Moreover, $z(\theta)$ is strictly increasing in ability.[17] Therefore, households with brighter children contribute more than households with less bright children. These results differ from De Fraja (2002), where households with higher income contribute less than those with lower income and households with less brighter children contribute more than those with brighter children (see Proposition 4, p.453).

4.3.1 The equilibrium when the government has access to a foreign market

By assuming that the government can freely transfer wealth between generations, we take a steady-state analysis (where by steady-state we mean constant distribution of taxes and educational benefits [18]). However, if we start from an economy that has not adopted this policy yet, then the old generation won't have agreed on a deferred payment $m_i(\theta)$ and we should ask the best way to implement this policy.[19] If there are enough resources to compensate for the lack of deferred payments in the first period then the answer is trivial. In this section, we show that if the government has access to a foreign credit market, then we also do not have any problems at implementing the optimal policy.

Define $D_i^t(\theta)$ as the government's deficit with the child whose ability is θ and parental income is Y_i at period t :

$$D_i^t(\theta) = \left(k - \hat{k}\right) e_i(\theta) - f \quad \text{at time } t$$

In that case, the government's intertemporal budget constraint is:

$$E\left[\sum_{t=0}^{\infty} \left(D_i^t(\theta) - t_i^t(\theta) + t_i^{t+1}(\theta)\right)\right] \leq 0$$

Assume that under the efficient education, the distribution of Y_i is already stationary (i.e. assume that the first generation has received the efficient amount of education). Since u and y are concave, and the government can transfer wealth linearly through the credit market, it follows that $D_i^t(\theta)$ must be constant in t. Hence, a necessary and sufficient condition for the government debt to be sustainable is $E[D_i(\theta)] \leq 0$ which is the budget constraint in the problem solved above.

Thus, when the government has access to a foreign market, the decentralized mechanism derived above achieves first-best and satisfies the government budget constraint. As $E[t_i(\theta)] = E[t_i^*(\theta)] > 0$, the government borrows from abroad to finance education and consumption. Each individual repays her loans in the following period.

[17] In the competitive equilibrium, $\hat{k} = \theta y_e\left(e_i^*(\theta); E\left[e_i^*(\theta)\right]\right) > 0$. Hence, since the educational level is increasing in θ, it follows that $z(\theta)$ is also increasing in θ.

[18] Since the distribution of the ability parameter is stationary, stationarity of education implies in stationarity of that wealth distribution.

As u and y are concave and the government can transfer wealth linearly across generations through taxes the optimal policy must be stationary.

[19] This point is similar to the discussions on the transition from pay-as-you-go to fully-funded systems of social-security in overlapping-generations models.

5 Conclusion

In this paper, we argue that the results obtained at De Fraja (2002) are a consequence of the assumption that the government's budget constraint must be balanced with each generation at every period. When the government is allowed to transfer wealth between generations, the results dramatically change. Since efficiency requires equality of marginal productivity of education across individuals and the optimal educational policies are Pareto efficient in this case, it follows that the amount of education does not depend on parental income (i.e. equality of opportunities in education is provided) .

The inefficiency of the laissez-faire equilibrium was due to two problems: imperfect credit markets and externalities generated by education. We have shown that the government should provide credit in order to correct the credit market inefficiency. Governmental provision of credit is probably the most suggested educational policy.[20] According to Becker (1991, p.188):

> 'Public (or private) policies that improve access to the capital markets by poorer families - perhaps a loan program to finance education (...) - would increase the efficiency of society's investments in human capital while equalizing opportunity and reducing inequality.'

By not internalizing the effects education causes on the rest of the economy, the amount of education each household provides in the laissez-faire equilibrium is inefficiently low. We show that the government may obtain the first-best solution through Pigouvian taxes. In this context, the appropriate Pigouvian taxes are educational subsidies that induce households to internalize for the (positive) externalities caused by education.[21]

Hence, the optimal mechanism can be decentralized through Pigouvian taxes and credit provision at the market interest rate. An advantage of decentralization is that it requires less information: the government may not know each household's wealth and the distributions of ability and wealth (It is sufficient to know the optimal externality level and the social marginal benefit it causes).

Moreover, each household's financial contribution does not depend on income and increases in the ability of the child. Thus, *the optimal educational policy is not regressive* (i.e., wealthier households do not contribute less than poorer households).[22]

[20] See, for example Barr (1993), Becker (1991), and Krueger and Bowen (1993). As Eden (1994) remarks: "Government backed loans can mitigate capital market imperfections and most economists will favor this type of intervention."

[21] Friedman (1955, pp.124-125) advocated for a scheme similar to the Pigouvian taxes proposed here. He argued that since buyers of education generate external benefits on those not purchasing education, the government should subsidize those purchasing education and tax those who are not.

[22] However, it is still *input regressive* and *output regressive* inasmuch as education and utility increase in ability (see Arrow, 1971).

A Appendix

Proof of Proposition (4):

Since (7) must hold with equality (because utility is increasing), we can substitute $m_i(\theta)$ in the welfare function:

$$W = \int_{\theta_0}^{\theta_1} \sum_{i=1}^{n} h_i \left[u\left(Y_i - \tau_i - f_i(\theta)\right) + y\left(\theta, e_i(\theta); E\right) + \tau_i + f_i(\theta) - ke_i(\theta)\right] \phi(\theta)\, d\theta$$

Substituting (4) in the above expression, it follows that:

$$W = \int_{\theta_0}^{\theta_1} \sum_{i=1}^{n} h_i \left[U_i(\theta) + m_i(\theta) + \tau_i + f_i(\theta) - ke_i(\theta)\right] \phi(\theta)\, d\theta$$

Introducing the auxiliary variable $S(\theta)$, (3) can be rewritten as:

$$\begin{aligned} \dot{S}(\theta) &= \sum_{i=1}^{n} h_i e_i(\theta)\, \phi(\theta) \\ S(\theta_0) &= 0,\ S(\theta_1) = E \end{aligned}$$

Then, the Hamiltonian to the government's problem is:

$$\begin{aligned} H \;=\; & \sum_{i=1}^{n} h_i \left[U_i(\theta) + m_i(\theta) + \tau_i + f_i(\theta) - ke_i(\theta)\right] \phi(\theta) + \\ & + \rho(\theta) \sum_{i=1}^{n} h_i e_i(\theta)\, \phi(\theta) + \gamma_i(\theta)\, y_\theta\left(\theta, e_i(\theta); E\right) + \\ & + \lambda_i(\theta) \left[u\left(Y_i - \tau_i - f_i(\theta)\right) + y\left(\theta, e_i(\theta); E\right) - m_i(\theta) - U_i(\theta)\right] + \\ & + \mu_i(\theta) \left[U_i(\theta) - P\left(\theta, Y_i - \tau_i, E\right)\right] \end{aligned}$$

The first order conditions are:

$$\frac{\partial H}{\partial m_i(\theta)} = 0 \therefore h_i \phi(\theta) = \lambda_i(\theta) \tag{15}$$

$$\frac{\partial H}{\partial f_i(\theta)} = 0 \therefore Y_i - \tau_i - f_i(\theta) = c^* \tag{16}$$

$$\frac{\partial H}{\partial \tau_i} = 0 \therefore \mu_i(\theta)\;\; P_Y\left(\theta, Y_i - \tau_i, E\right) = 0 \therefore \mu_i(\theta) = 0 \tag{17}$$

$$\frac{\partial H}{\partial e_i(\theta)} = 0 \therefore -kh_i\phi(\theta) + \rho(\theta)\, h_i \phi(\theta) + \gamma_i(\theta)\, y_{\theta e}\left(\theta, e_i(\theta); E\right) + \lambda_i y_e\left(\theta, e_i(\theta); E\right) = 0 \tag{18}$$

$$\frac{\partial H}{\partial U_i(\theta)} = -\dot{\gamma}_i(\theta) \therefore \gamma_i(\theta) = \gamma_i \text{ constant for all } \theta \tag{19}$$

$$\frac{\partial H}{\partial S(\theta)} = -\dot{\rho}(\theta) \therefore (\theta) = \rho \text{ constant for all } \theta \tag{20}$$

Lemma 7 *$\gamma_i = 0$ for all $i \in \{1, ..., n\}$.*

Proof. As $U_i(\theta_1)$ is free, the transversality condition is $\gamma_i(\theta_1) = 0$ for all i. Hence, (19) implies in $\gamma_i = 0$, $\forall i \in \{1, ..., n\}$. ■

Substituting $\gamma_i = 0$ in equation (18), it follows that:

$$y_e(\theta, e_i(\theta); E) = k - \rho \tag{21}$$

Lemma 8 *The amount of second-best education is the same as in the first-best solution. That is, $e_i(\theta) = e^*(\theta)$, $\forall i \in \{1, ..., n\}, \forall \theta \in [\theta_0, \theta_1]$.*

Proof. $\frac{\partial W}{\partial E}\Big|_{E=E^{2b}} = \rho \therefore \int_{\theta_0}^{\theta_1} \sum_{i=1}^{n} h_i y_E\left(\theta, e_i^{2b}(\theta), E^{2b}\right) \phi(\theta)\, d\theta = \rho$. Substituting in (21), it follows that:

$$y_e(\theta, e_i(\theta); E) = k - E\left[y_E\left(\theta, e_i^{2b}(\theta), E^{2b}\right)\right]$$

which is the equation that implicitly defines $e_i^*(\theta)$. ■

Hence, the amount of education and consumption provided at the optimal educational policy is the same as in the first-best solution. Since individuals utilities are linear in deferred payments $m_i(\theta)$, it follows that any profile of deferred payments such that the government's budget constraint is satisfied as an equality achieves the same welfare W. Therefore, first-best welfare is reached in the second-best equilibrium. Moreover, we have already shown that $e^*(\theta)$ is increasing in θ implying that this policy is incentive compatible (since it also satisfies the monotonicity condition: $\dot{e}(\theta) \geq 0$).

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